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Estimates for Fourier sums and eigenvalues of integral operators via multipliers on the sphere

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Resumo

This work intends to provide decay rates for the sequence of eigenvalues of positive integral operators generated by kernels satisfying an abstract Hölder condition defined by a class of multipliers operators. For such purpose, the work involves the deduction of convenient estimates for the Fourier coefficients of integrable functions on the esphere through the rate of approximation of the class of multipliers operators that we will work with.

Let S^m denote the m-dimensional unit sphere in the euclidian space $\mathbb{R}^{m+1}$, endowed with the usual Lebesgue measure σ_m. We denote by ω_m the surface area of S^m. In this work, we will deal with the usual spaces $L^p(S^m) := L^p(S^m, \sigma_m)$, the norm of which we denote by $\| \cdot \|_p$. A multiplier operator refers to a linear operator T on $L^p(S^m)$ for which there exists a sequence $\{\eta_k\}$ of complex numbers (called the sequence of multipliers of T) such that $\mathcal{Y}_k(Tf) = \eta_k \mathcal{Y}_k(f)$, $f \in L^p(S^m)$ and $k = 0, 1, \ldots$. An important category of bounded multiplier operators are those given by a the convolution with a zonal measure. The class of bounded multiplier operators on $L^1(S^m)$ was characterized by C. Dunkl as that composed of operators which are convolutions with zonal measures on S^m. Among other things, this characterization reveals that the class of bounded multiplier operators on $L^2(S^m)$ is bigger than that of bounded multiplier operators on $L^1(S^m)$. Also, it is not hard to see that a multiplier operator on $L^2(S^m)$ is bounded if and only if its sequence of multipliers is bounded.

We first consider a family of multipliers operators $\{M_t :\in (0, \pi)\}$ acting on $L^2(S^m)$. We deduce an estimate for certain sums of Fourier coefficients of integrable functions on the sphere. Estimates of this sort can be found in [1] and it gives us a control of the growth of these coefficients by the rate of approximation of $\{M_t :\in (0, \pi)\}$. After that, we introduce a Hölder condition attached to it as follows.

We say that a a kernel K in $L^2(S^m \times S^m) := L^2(S^m \times S^m, \sigma_m \times \sigma_m)$ is $\{M_t : t \in (0, \pi)\}$-Hölder if there exist a real number $\beta \in (0, 2]$ and a constant $B > 0$ so that

$$\int_{S^m} |M_t(K^y)(y) - K^y(y)| d\sigma_m(y) \leq Bt^\beta, \quad t \in (0, \pi). \tag{1}$$

The above Hölder condition is implied by the more classical one which demands the existence of $\beta \in (0, 2]$ and a function B in $L^1(S^m)$ so that $\sup_x |M_t(K^y)(x) - K^y(x)| \leq B(y)t^\beta$, $y \in S^m$, $t \in (0, \pi)$.

Using a technique introduced in [2], the goal here is to deduce decay rates for certain positive integral operators on the sphere, those generated by a Mercer-like kernel satisfying a Hölder condition defined by a parameterized family of multipliers operators on $L^2(S^m)$, as that defined in (1). The main contribution of present work brings an important advance: the use of an abstract Hölder condition coupled with an abstract setting. In particular, many other settings can be putted into that of this note, and important classical results in the literature can be easily recovered ([2,3]).

Referências

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